Euclidean and non-Euclidean Geometry (MA3101)
Lecture 2: Perspective

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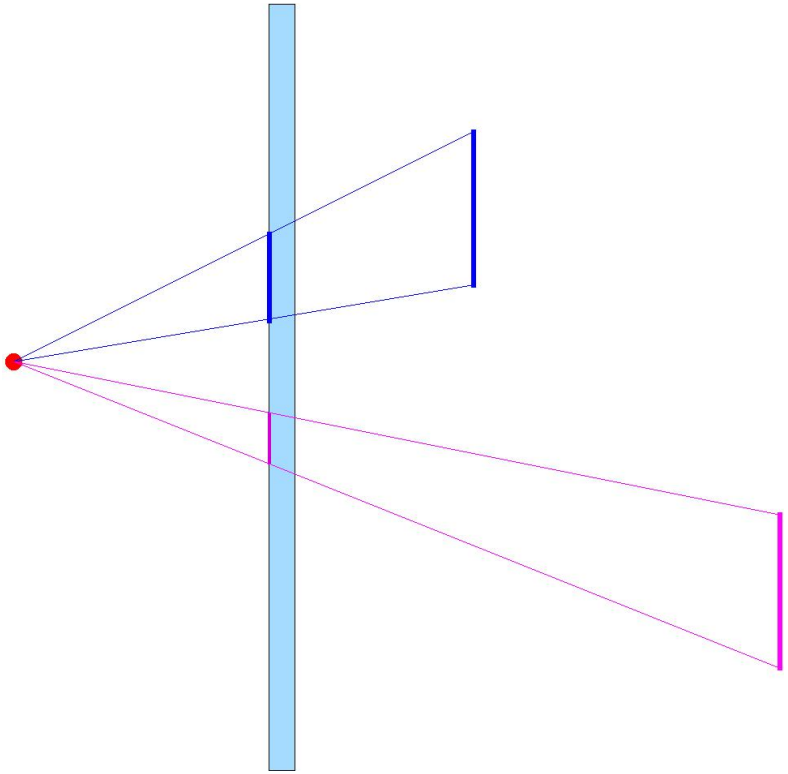
September 11, 2024

Lecture 2: Perspective

Why do objects that are further away look smaller?



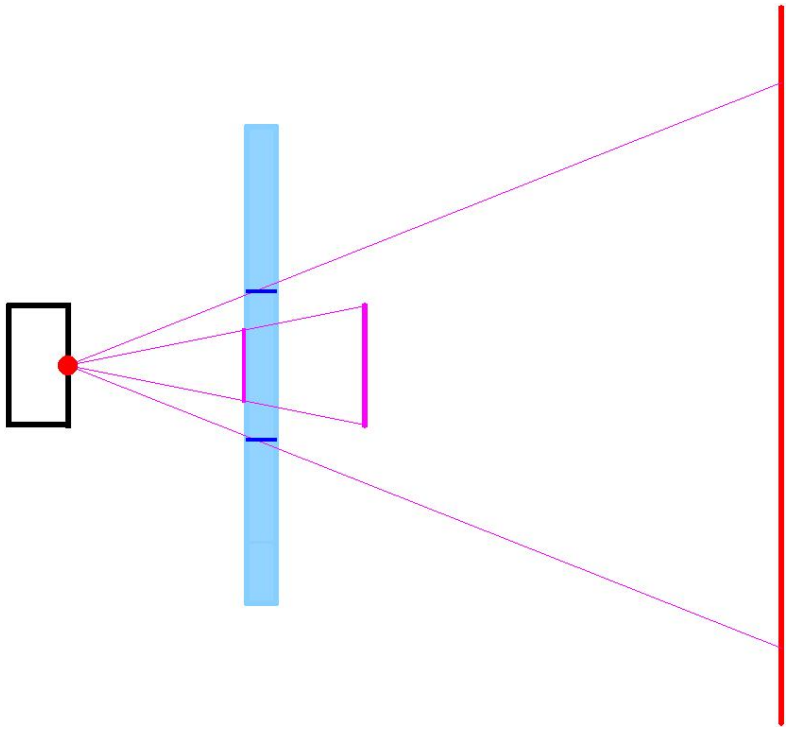
- Red dot: viewer's eye
- Blue vertical: the window.

The pink and blue objects are identical. Their images on the window show how they are perceived by the viewer.

Note The window doesn't change what is seen. It just provides an explicit plane on which the scene is projected.

Optical effects - the "dolly zoom" (Goodfellas, 1990)

You are making a movie. Your protagonist is in the foreground (pink). You want them to occupy half of the height of the frame. You want viewers to have the sense that the background is closing in on them. You can move the camera forward and back. And you can adjust the zoom.



- Red dot: camera
- Red line: background
- Dark blue: top and bottom of frame
- Between pink lines: background visible in frame

What should you do for the desired effect?

Optical effects - the "dolly zoom" (Goodfellas, 1990)

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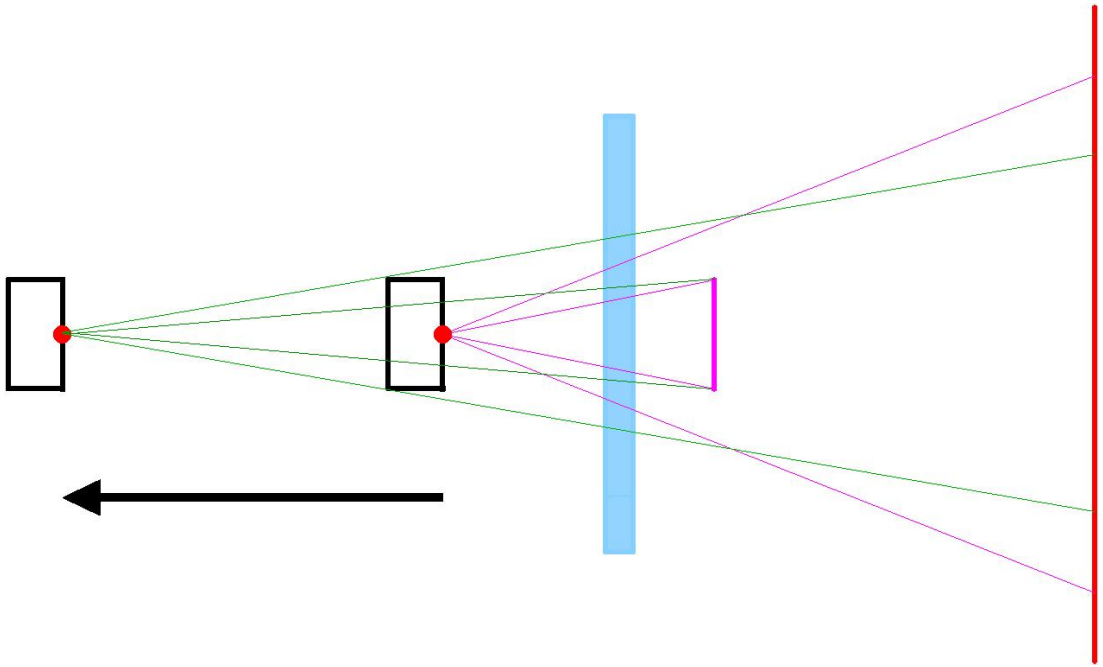

Slide the camera backwards!

Optical effects - the "dolly zoom" (Goodfellas, 1990)

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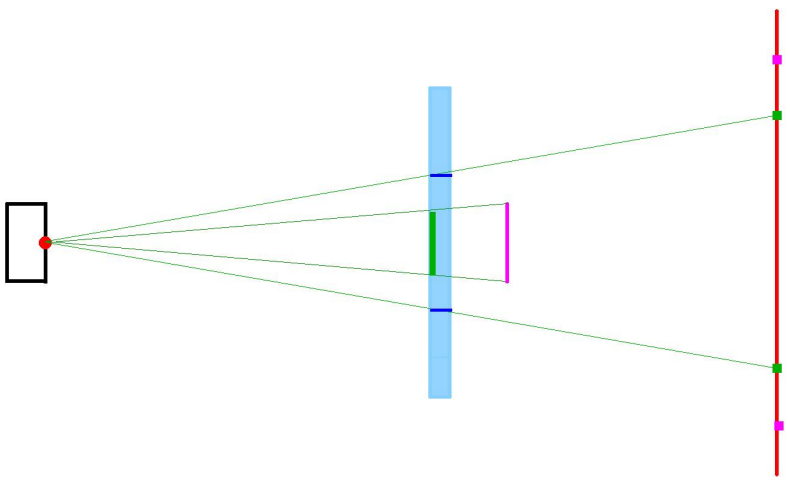
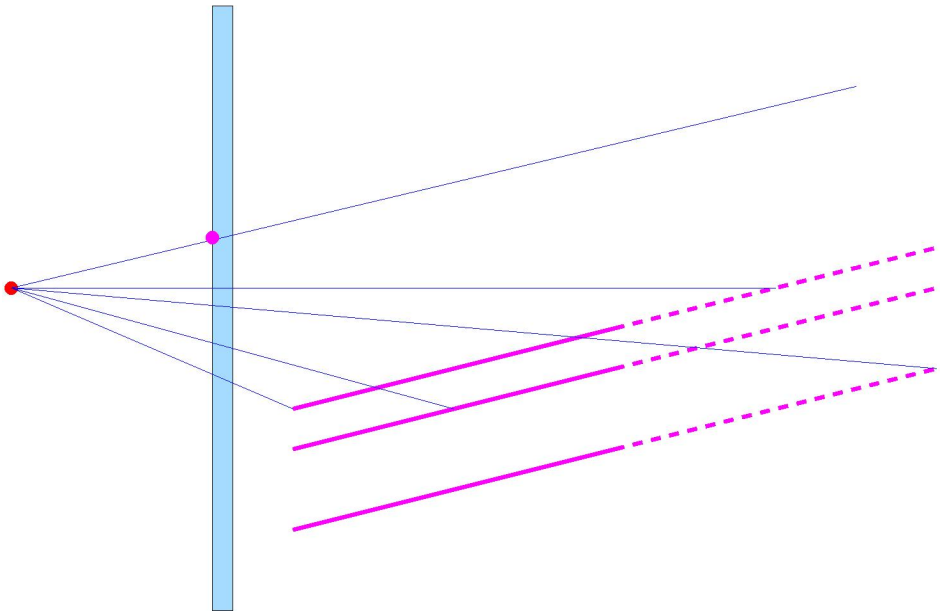


Protagonist occupies half of vertical frame.
Between pink marks: original visible background.
Between green marks: final visible background - still occupies the whole frame so it looks bigger and closer!

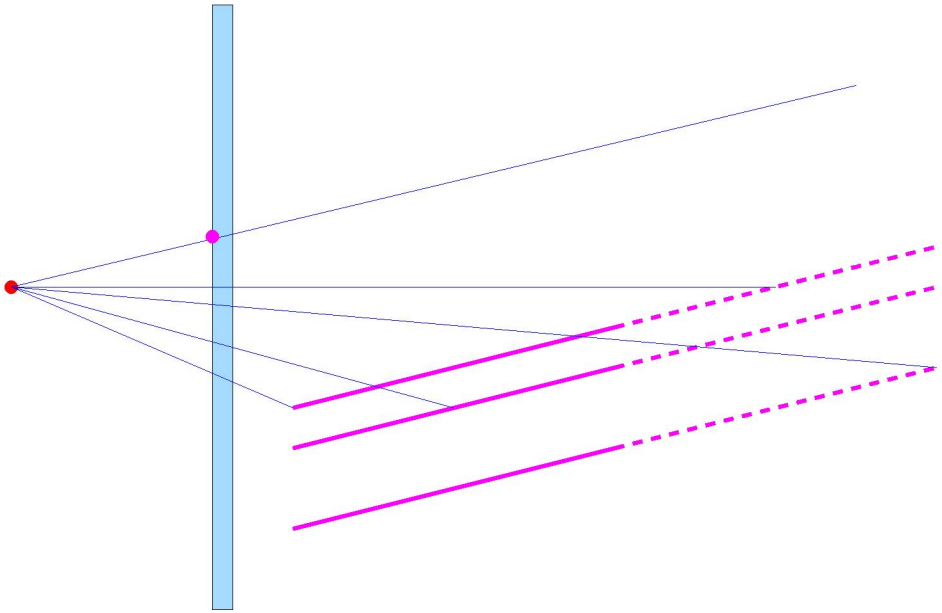
Back to the question from Lecture 1

If lines outside are parallel to each other, (when) are their (taped) images parallel on the window?
If the taped images are not parallel, where is their intersection point on the window (in relation to the position of the viewer's eye?)



- Solid pink: parallel lines outside
- Dotted pink: "imaginary" continuation
- Blue: lines of sight as viewer raises head.

Back to the question from Lecture 1

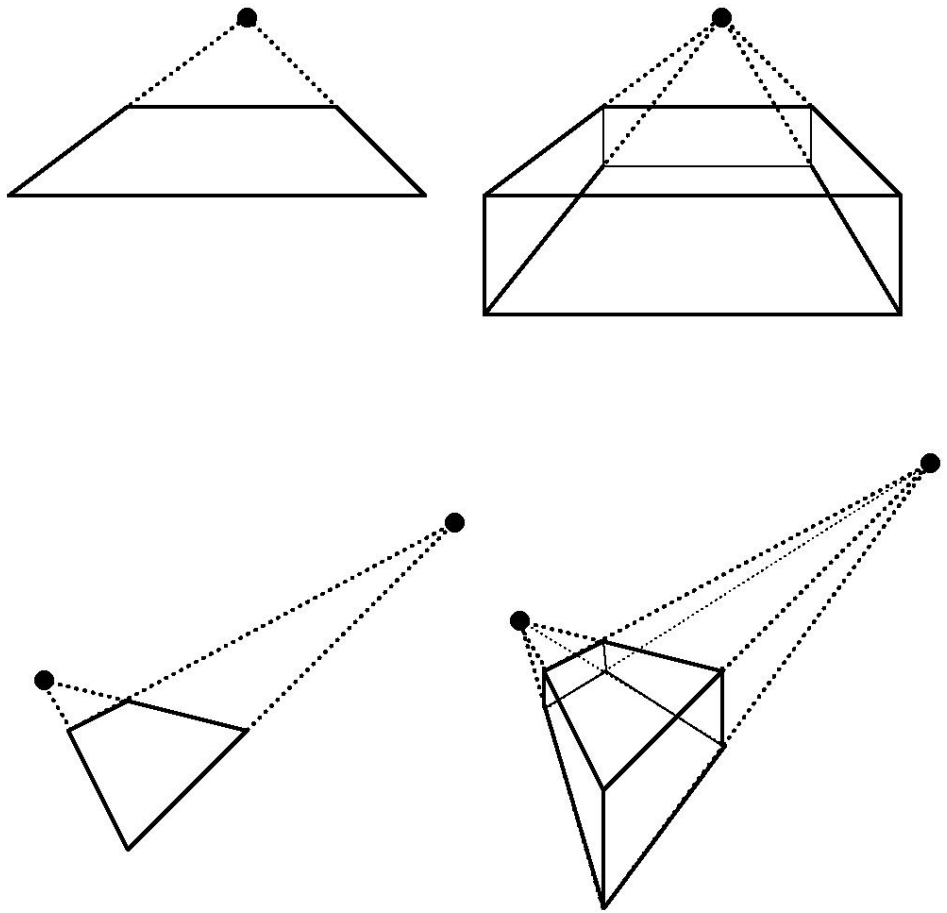


- Solid pink: parallel lines outside
- Dotted pink: "imaginary" continuation
- Blue: lines of sight as viewer raises head.

If the viewer raises her head until her line of sight is parallel to the pink lines, she can no longer see them or their continuation. The pink dot is the intersection of this line of sight with the window (the "picture plane"). This is the intersection point of the images on the glass of the pink lines.

One-point and two-point perspective pictures

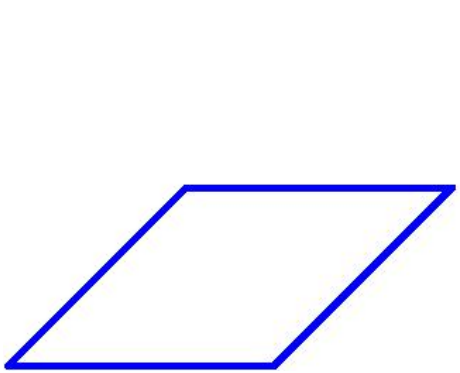
In one-point perspective, there is a single direction of parallel lines that are not parallel to the picture plane ("window"). There is a single vanishing point for all lines in this direction. In two-point perspective, there are two vanishing points for different sets of parallel lines.

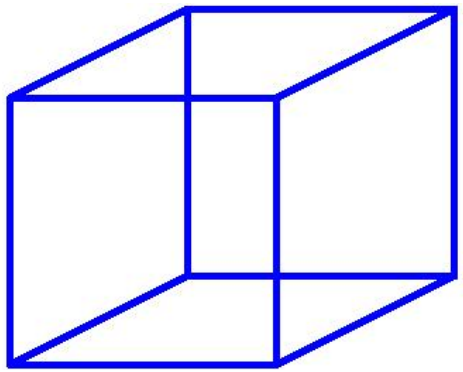


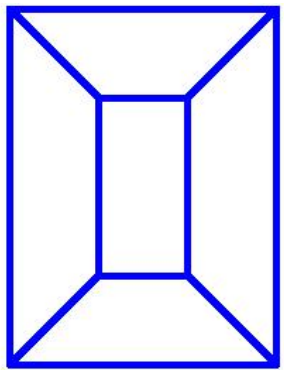
One-point and two-point perspective drawings of a rectangle and retangular box.

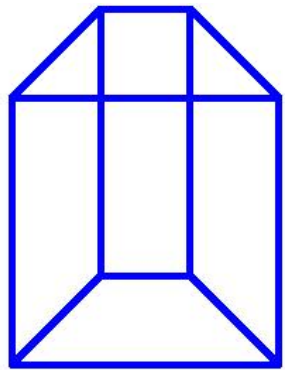
Question

Could any of the following be one-point perspective pictures of a rectangle or a rectangular box?









A look ahead to projective geometry

A viewer looks at $\mathbb{R}^3$ (their surroundings) through a (infinite) window. Their eye is at the origin O. The window is a plane. Every point of the window represents a line through O in $\mathbb{R}^3$. Every *line* in the window represents a plane through O in $\mathbb{R}^3$. The window is a "model" for the real projective plane.