MA3101 EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY 2024-2025
**Assignment 1**
Due date: Wednesday October 16

Problems marked with ∗ are for submission. The rest are for independent study and discussion in the tutorials. Please submit solutions via the Canvas page, or in person at the lecture if you prefer paper.

1. In the picture below, the blue line indicates a window. A person is standing somewhere on the black line looking through the window at a rectangle that is marked on the ground outside. The blue quadrilateral shows the outline of the rectangle as it appears to the person (this is what they would draw on the window if they trace the rectangle as they see it).
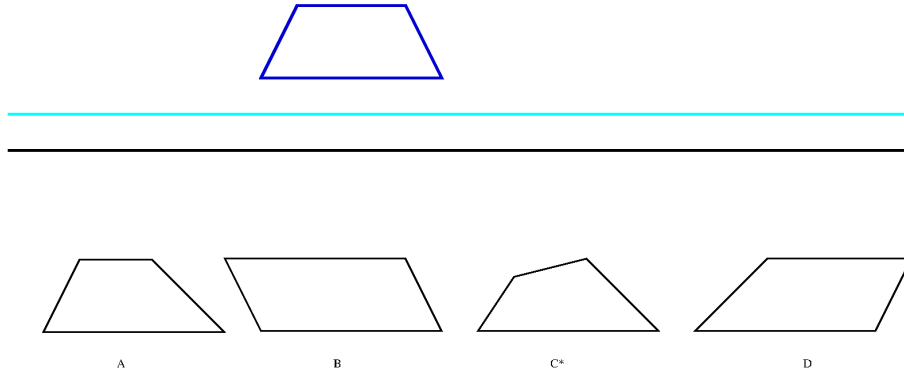
   For each of the pictures A,B,C,D, state whether it *could* be the view that the person would have of the rectangle from another position on the black line. In all cases where you answer YES, there is no need to give an explanation. If you say NO, give a short explanation why not.

2. For each of the following pictures, explain why it is *not* an accurate perspective drawing of a rectangular box.
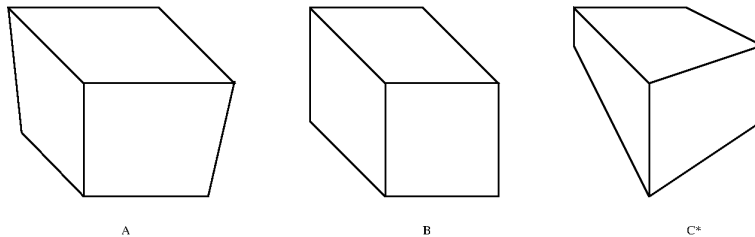
3. A *torus* is the surface obtained by rotating a circle about an axis that does not intersect the circle. Describe three different examples of geodesics on the surface of a torus, including one that is not a closed curve and two that are circles.

4. * Let P be a plane in $\mathbb{R}^3$ (not necessarily passing through O) and suppose that $P \cap S^2$ contains at least two distinct points. Show that $P \cap S^2$ is a circle.

5. Find the radius and the centre of the (Euclidean) circle in $\mathbb{R}^3$ that is the intersection of S^2 with the plane $P : x + y + z = 1$.

6. * Give a parametric description of the circle of intersection of S^2 and the plane $z = \frac{1}{2}$.

7. * The 52° parallel of latitude passes through County Cork and the 55° parallel passes through County Donegal. Determine the length (in km) of an east to west journey through one degree of longitude at $52°N$ and at $55°N$. Take the Earth to be a sphere of radius 6,500km.

8. Find the two common points of intersection of the sphere S^2 and the planes with equations $x + 2y - 3z = 0$ and $x - y - 2z = 0$.

9. * Use the spherical cosine rule to find the distance on the Earth's surface from Galway (53.3°N,9.1°W) to Rio de Janeiro (22.9°S,43.2°W). Take the Earth to be a sphere of radius 6,500km.

10. * Let P and Q be distinct points of S^2.

    (a) Prove that the set of points in $\mathbb{R}^3$ that are equidistant from P and Q (in terms of the Euclidean distance in $\mathbb{R}^3$) intersects S^2 in a great circle.

    (b) Prove that this great circle is the set of points of S^2 that are equidistant from P and Q in the intrinsic geometry of S^2. (In the intrinsic geometry of S^2, the distance between two points is the length of the shortest great circle arc between them).

    (c) Give a parametric desciption of this great circle in the case $P = (1, 0, 0), Q = (0, 0, 1)$.