MA283: Linear Algebra
Semester 2 2022-23
Practice Problem Sheet 1

1. Apply a sequence of elementary row operations to the matrix A below, to obtain (i) a row echelon form, and (ii) a reduced row echelon form.

$$A = \begin{pmatrix} 2 & 2 & 1 & -1 & 7 \\ 2 & 1 & -3 & -6 & 15 \\ 4 & -1 & 1 & -8 & -3 \end{pmatrix}$$

2. Find the general solution of the following system of linear equations.

$$\begin{array}{rcrcrcrcr} 2x & + & 2y & + & z & - & w & = & 7 \\ 2x & + & y & - & 3z & - & 6z & = & 15 \\ 4x & - & y & + & z & - & 8w & = & -3 \end{array}$$

3. Using your answer to 2. above, show that the system

$$\begin{array}{rcrcrcrcr} 2x & + & 2y & + & z & - & w & = & 7 \\ 2x & + & y & - & 3z & - & 6w & = & 15 \\ 4x & - & y & + & z & - & 8w & = & -3 \\ x & + & 2y & + & z & + & w & = & 1 \end{array}$$

is inconsistent.

4. Use your answer to 2. above to find the unique solution of the system

$$\begin{array}{rcrcrcrcr} 2x & + & 2y & + & z & - & w & = & 7 \\ 2x & + & y & - & 3z & - & 6z & = & 15 \\ 4x & - & y & + & z & - & 8w & = & -3 \\ 2x & + & 2y & + & z & + & w & = & 1 \end{array}$$

5. Answer TRUE or FALSE to each of the following.

   (a) A system of linear equations with more variables than equations must have infinitely many solutions.

   (b) A system of linear equations with more variables than equations cannot have a unique solution.

   (c) A system of linear equations with a unique solution must have the same number of variables as equations.

   (d) A system of linear equations with fewer equations than variables is always consistent.

   (e) If a linear system has infinitely many solutions, then a reduced row echelon form obtained from its augmented matrix has at least two columns without a leading 1.

6. The following two tables show census data from 1960, 1970 and 1980, from the regions of Dedekind Valley (DV), Schur Hills (SH) and Taussky Town (TT). The first table shows the number of households in each region recorded in each of these years, and the second shows the mean number of people living in a household in each of the three regions in each year.

	DV	SH	TT
1970	5000	8500	10000
1980	4500	7500	10000
1990	6000	10000	15000

	DV	SH	TT
1970	3.2	3.8	2.5
1980	3.0	3.5	2.5
1990	3.0	4.0	2.0

Let A and B denote these tables, interpreted as 3×3 matrices. What interpretation would you give to the entries of the matrix AB^T? What about $A^T B$?

7. Let $A = \begin{pmatrix} 2 & -3 & 4 & 1 \\ -1 & 3 & 0 & -1 \\ 2 & -2 & 5 & 3 \end{pmatrix}$.

   Find the value of each of the following expressions.

   (a) $\sum_{j=1}^{4} A_{1j}$ (b) $\sum_{i=1}^{3} A_{i3}$ (c) $\sum_{i=1}^{3} A_{ii}$ (d) $\sum_{i=1}^{3} A_{i2}A_{i3}$ (e) $\sum_{j=1}^{4} A_{2j}A^{T}_{j3}$

8. Calculate the matrix products AB and BA, where

$$A = \begin{pmatrix} 1 & 4 & 0 \\ -2 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -3 \\ 4 & -1 \\ -2 & -2 \end{pmatrix}.$$

9. For the matrix A of Question 8 above, calculate the products AA^{T} and $A^{T}A$.