MA3343 Groups

Introduction - Lecture 1

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Group Theory and Abstract Algebra

Algebra is the study of algebraic structures. An algebraic structure is a set of objects (e.g. numbers, functions, matrices) with a binary operation.

A binary operation is a way of combining any pair of the objects in the set to produce a new object in the same set (e.g. addition of integers, multiplication of 2×2 matrices, composition of functions from $\mathbb{R}$ to $\mathbb{R}$).

It's possible to study the properties of binary operations on sets abstractly, without particular attention to the elements to which they apply. Different combinations of properties determine different types of algebraic structures, like vector spaces, groups, rings, fields and many others.

Planned structure of this module

Resources are at `http://www.rkq.ie/teaching/ma3343`.

- Monday/Tuesday - weekly online content posted, and weekly email update

  Weekly content includes lecture notes and backup videos
- Thursday 12.00 (AC202) and Friday 12.00 (AC202) - lectures. At lectures we will
  - discuss current content;
  - look at homework problems;
  - work on poster projects.
- Tutorials will begin in Week 3 or 4.

Assessment - flexible with several elements

1. Two homework assignments, in Week 3 and Week 6 approx, worth 15% each.
2. Group poster project - team up with classmates and create a poster on a topic in group theory for our end of year exhibition in December (20%).
3. Final exam in the December exam session (70%).

This adds up to more than 100%. Marks above 100 will be rounded down. You don't have to do all the components, or any particular one.

Secton 1.1 Examples of Groups

1. $(\mathbb{Z}, +)$
2. $(\mathbb{C}^\times, \times)$
3. $(\mathrm{GL}(2, \mathbb{Q}), \times)$ Read this as "the general linear group of 2 by 2 matrices over the rational numbers" ("GL" stands for "general linear"). This time, $\times$ denotes matrix multiplication.
4. $(\{1, i, -i, -1\}, \times)$
5. Let S_3 denote the following set of 3×3 matrices.

$$S_3 = \left\{ \begin{array}{ccc} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\ \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, & \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \right\}.$$