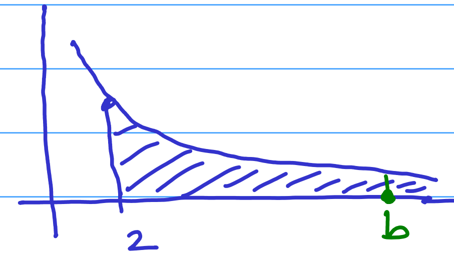
March 11 A couple of extra examples of improper integrals

(1) $\displaystyle\int_2^{\infty} \frac{1}{\sqrt{x^3}}\,dx$



$$\int_2^b \frac{1}{\sqrt{x^3}}\,dx = \int_2^b x^{-3/2}\,dx$$

$$= -2x^{-1/2}\Big|_2^b = -2/\sqrt{x}\,\Big|_2^b = -2/\sqrt{b} + 2/\sqrt{2}$$

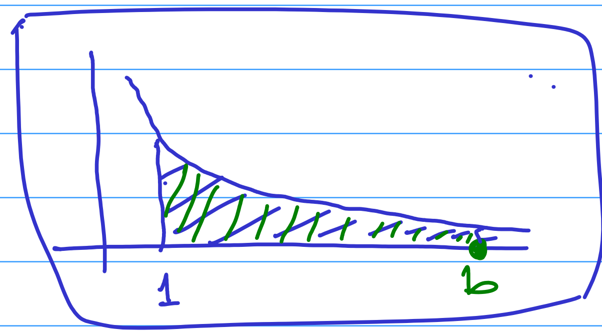
$$= \sqrt{2} - 2/\sqrt{b}$$

$$\lim_{b\to\infty}\left(\sqrt{2} - \underbrace{\cancel{2/\sqrt{b}}}_{\to 0 \text{ as } b\to\infty}\right) = \boxed{\sqrt{2}}$$

Conclusion

$\displaystyle\int_2^{\infty} \frac{1}{\sqrt{x^3}}\,dx$ converges and its value is $\sqrt{2}$.

(2) $\displaystyle\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$



$$\int_1^b \frac{1}{\sqrt{x}}\,dx = \int_1^b x^{-1/2}\,dx$$

$$= 2x^{1/2}\Big|_1^b = 2\sqrt{x}\,\Big|_1^b$$

$$= \boxed{2\sqrt{b} - 2}$$

$$\lim_{b\to\infty}\left(2\sqrt{b} - 2\right)$$

$2\sqrt{b} - 2 \to \infty$ as $b \to \infty$

The limit does not exist, the improper integral is divergent and the region has infinite area.