1. $$\int_0^{\sqrt{3}} \frac{r}{\sqrt{1+r^2}}\,dr$$

Try $u = 1+r^2$. $\frac{du}{dr} = 2r \Rightarrow du = 2r\,dr$, $r\,dr = \frac{1}{2}du$

Limits: $r=0 \rightarrow u = 1+0^2 = 1$ $\quad r=\sqrt{3}$ $u = 1+(\sqrt{3})^2 = 4$

$$\frac{1}{2}\int_{u=1}^{u=4} \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_1^4 u^{-1/2}\,du = \frac{1}{\cancel{2}}\cdot\cancel{2}u^{1/2}\Big|_{u=1}^{u=4}$$

$$= \sqrt{u}\Big|_{u=1}^{u=4} = \sqrt{4}-\sqrt{1} = 2-1 = \boxed{1}$$

2. $$\int_0^1 x\,e^{2x}\,dx$$

Integration by parts $\quad u = x \quad v' = e^{2x}$

$u' = 1 \quad v = \frac{1}{2}e^{2x}$

$$\int uv'\,dx = uv - \int u'v\,dx \qquad \int_0^1 x e^{2x}\,dx = \frac{1}{2}xe^{2x}\Big|_0^1 - \frac{1}{2}\int_0^1 e^{2x}\,dx$$

$$= \frac{1}{2}x e^{2x}\Big|_0^1 - \frac{1}{2}\left[\frac{1}{2}e^{2x}\right]\Big|_0^1$$

$$= \left(\frac{1}{2}e^2 - \cancel{\frac{1}{2}\cdot 0\cdot e^0}\right) - \left(\frac{1}{4}e^2 - \frac{1}{4}\right)$$

$$= \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4} = \frac{1}{4}e^2 + 1/4$$

3. $$\int_{\pi/2}^{\pi} \sin^2 x\,\boxed{\cos x}\,dx$$

Try $u = \sin x$ $\quad \frac{du}{dx} = \cos x \quad du = \cos x\,dx$

Limits $x = \pi/2$, $u = \sin(\pi/2) = 1$ $\quad x = \pi \quad u = \sin\pi = 0$

$$\int_1^0 u^2\,du = u^3/3\Big|_1^0 = 0 - 1/3 = -1/3$$